**TECHNIQUES FOR CALCULATING LIMITS**

1. **Substitution**

Calculate .

* When substituting 3 into x, we get the value = = -2.
* Therefore = -2.

1. **Simplify algebraically before substitution**
2. Calculate .

* When substituting 1 into x, we get the value = which is undefined.
* Evaluate the limit again by factorising and simplifying before substitution.
* = = .
* Now when substituting 1 into x we get the value 2.
* Therefore = 2.

1. Calculate .

* Direct substitution: = . Evaluate again.
* = = .
* Substitute x = -1: = = -0.5
* Therefore = -0.5.

When substituting values into x, the general guide to determining the limit is:

|  |  |
| --- | --- |
| Result when substituting | Conclusion |
| sensible, defined value | the value is the limit |
| e.g. | undefined, no limit |
| e.g. | limit is 0 |
|  | factorise, simplify, substitute again |

1. **For the case of x 0**

If substitution, or factorising before substitution, does not work -> Substitute a small number into x,

and evaluate on a calculator.

If substituting -0.0001 into x produces a value different to 1, then no limit exists.

1. Calculate .

* When substituting 0 into x, we get the value .
* Function cannot be factorised.
* Substitute a small number like 0.0001 into function: = 0.99999.... = 1
* Therefore = 1.

1. Calculate .

* Direct substitution: is undefined.
* Function cannot be factorised.
* Substitute a small number like 0.0001 into function: = 0.999.... = 1
* Therefore = 1.

1. **For the case of x**

**Either:**

Divide each term in the numerator and denominator by x.

The rule , where is a constant, will be useful.

1. Calculate .

* Divide all terms by x:
* = = 1 + 0 = 1.
* Therefore = 1.

1. Calculate .

* Divide all terms by x:
* = .
* Therefore = 2.

**Or:**

Substitute a large number into x, and evaluate on a calculator.

1. Calculate .

* Substitute a large number like 10000 into function: .
* Therefore = 1.

1. Calculate .

* Substitute a large number like 10000 into function: .
* Therefore = 2.